

# MAGNETIC FRACTAL DIMENSIONALITY OF THE DIELECTRIC BREAKDOWN UNDER STRONG MAGNETIC FIELDS

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The formation of breakdown pattern on an insulating surface under the influence of a transverse magnetic field is theoretically investigated. We have generalized the Dielectric Breakdown Model (DBM) for the case of external magnetic field. Concept of the Magnetic Fractal Dimensionality (MFD) is introduced and its universality is demonstrated. It is shown that MFD saturates with magnetic fields. The magnetic field dependence of the streamer curvature is obtained. It is conjectured that nonlinear current interaction is responsible for the experimentally observed 'spider-legs' like streamer patterns.

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Fractal properties are common to the dielectric breakdown phenomena [1] which range from an atmospheric lightning [2],[4], [3] to electric treeing in polymers [5] and are of scientific and technical importance [6],[7]. Although the actual physical processes can be quite different in these phenomena, the global properties of the resulting discharge patterns are very similar. Filamentary gas discharges on insulating surface exhibit remarkable similarities to breakdown phenomena in long gaps [8], e.g., to atmospheric lightning, and thus offer the possibility to perform well-defined model experiments in laboratory. The application of sophisticated diagnostic tools, such as streak cameras, high-speed oscilloscopes, and time-resolved spectroscopy [9], in surface discharge experiments has improved the basic understanding of how a highly conducting phase, the filamentary "leader" channel, advances into a nonconducting medium as the surrounding gas.

The surface discharge in compressed  $SF_6$  gas have been studied in detail by Niemeyer and Pinnekamp [10]. The parameters were controlled in such a way that the experiment produces, to a good approximation, an equipotential channel system growing in a plane with a radial electrode from a central point. The experiment shows that the dielectric breakdown pattern has a fractal structure.

The stochastic model containing the essential features of the fractal properties of the dielectric breakdown was introduced by Niemeyer, Pietronero, and Wiesmann (NPW) [11],[1]. The introduced stochastic model made it possible to simulate the growth of a fractal structure which resembles a Lichtenberg figure [2]. The basic assumption of the dielectric breakdown model (DBM) was that the growth probability depends on the local field. This model naturally leads to fractal structures. In [11] the Hausdorff dimension and other fractal properties of the structure are determined and their close relations to the other fractal structures e.g., DLA (the diffusion limited aggregation) [12], is shown.

The DBM method was generalized by Wiesmann and Zeller [13] in two ways. Firstly they introduced a critical field for growth  $F_c$ . The growth probability was assumed to be proportional to the local field  $F_{loc}$  if  $F_{loc} > F_c$  and zero if  $F_{loc} < F_c$ . Secondly they have introduced an internal field in the structure  $F_s$ . The potential in the structure was no longer equal to the potential  $V_0$  of the connecting electrode but equal to  $V_0 + F_s \cdot s$ , where  $s$  is the length of the path along the structure which connects the point to the electrode.

In the recent experiment [3] a transverse high magnetic field was applied during the discharge evolution and thus any spatial restriction of the surface discharge was avoided in order to use a locally sensitive probe.

When a rectangular high-voltage pulse of about  $500ns$  length,  $20kV$  amplitude, and less than  $10ns$  risetime is applied to a point-to-plane electrode system, the electrodes being separated by a thin dielectric film (thickness  $\approx 100\mu m$ ), a discharge propagates in the gas just above the surface of the film. The discharge pattern was recorded directly with high resolution by using a photographic film as dielectric plate.

The experiment shows the spatial evolution of a negative surface discharge in a nitrogen atmosphere as a function of the magnetic field (Fig.2 of Ref. [3]). At  $B = 0$  a very bright starlike pattern develops. At moderate magnetic field (up to  $7T$ ), the leader channels are bent and appear to have a circular shape outside the central electrode region.

The radius of curvature is of the order of  $1cm$  at  $7T$ . The direction of the bending corresponds to the movement of electrons in crossed electric and magnetic fields. With the increase of the magnetic field the radius of curvature decreases, the channels approach each other and branching sets in. At the highest applied magnetic field of  $12T$ , circular-shaped current filaments are only found in the outmost regions of the discharge pattern where they can develop undistributed by the fields of neighboring leader channels.

One can summarize the experimental results as follows:

(1) The observed bending effect cannot be related to the movement of a single charged particles in crossed electric and magnetic fields, which should result in the curvatures of the order of magnitude of  $15\mu m$  at  $1T$  for the electrons, i.e., the Larmor radius  $r_c = v_c/\omega_c$  of the cyclotron orbit of an electron with enough kinetic energy to ionize gas molecules. It has been assumed that the macroscopic value of the radius of curvature results from the drift movement of the Larmor centers of the gyrating electrons. The average radius of curvature follows a power law,  $R \propto B^{-\alpha}$ . The exponent  $\alpha$  depends on the nature of gas.

(2) The circular shape of the current filaments is not expected to occur for our point-to-plane electrode geometry.

(3) The complexity of the discharge pattern can be controlled by the magnetic field strength. It appears

that an increase in complexity is induced on the scale of the bending observed, and not on the microscopic scale of the cyclotron radius.

In what follows we modify the model introduced by Wiesmann and Zeller [13] for the case of external magnetic field.

Let us consider a two-dimensional square lattice in which a central point represents one of the electrodes while the other electrode is modeled as a circle at large distance, which represents the geometry of the experiment. The discharge starts at the central electrode and grows by one lattice bond per growth step. A bond connects a lattice point with one of eight adjacent lattice points as it is described in Fig.1. Once a given point is connected to the discharge structure by a bond, it becomes part of the structure. The potential of the central electrode is 0, the potential of a point in the structure is  $V_{i,k} = V_{l,m} + V_R * l$ , where  $V_{l,m}$  is the potential of a points from which growth go on,  $V_R$  is an internal field in the structure and  $l$  is 1 for bonds parallel to the grid and is  $\sqrt{2}$  for diagonal bonds.

The growth is computed as follows: First the Laplace equation is solved with the boundary conditions determined by the electrodes and the discharge structure. Then the local field  $F_{loc}$  between a point which is already a part of the structure  $(i, k)$  and a new adjacent point  $(i', k')$  is calculated:

$$F_{loc}(i, k, i', k') = \frac{\varphi_{i', k'} - V_{i, k}}{l} \quad (1)$$

where  $\varphi_{i', k'}$  is the solution of Laplace equation. The breakdown can occur only if the local field is greater than the critical field of growth  $F_C$ . The probability that a new bond will form between a point which is already a part of the structure and a new adjacent point  $p(i, k \rightarrow i', k')$  is calculated as a function of the local field  $F_{loc}$  between the two points:

$$p(i, k \rightarrow i', k') = \begin{cases} 0 & , \quad F_{loc}(i, k, i', k') < F_C \\ \frac{F_{loc}^\eta}{\sum F_{loc}^\eta} & , \quad F_{loc}(i, k, i', k') \geq F_C \end{cases} \quad (2)$$

where a power-law dependence with exponent  $\eta$  is assumed to describe adequately the relation between the local field and the probability. The sum in the denominator refers to all possible growth processes. A new bond is chosen randomly with probability distribution (2) and added to the discharge pattern. With this new discharge pattern one starts again. More detailed description of this model is done in [13].

Let us take into account the magnetic field. The magnetic field changes the probability distribution (2). A moving particle in the magnetic field experiences the Lorentz force  $F_L \sim \vec{V} \times \vec{H}$  which acts perpendicularly to its velocity. Consider each step of growth like a superposition of two processes. The first step is choosing a new bond using the probability distribution (2), and the second step is taking into account the probability of deviation of the growth due to the magnetic field,  $p_H$ .

If after first step of growth, for example, the bond from the dot 0 to the dot 4 was chosen (Fig.1), the growth will occur to the dot 3 with the probability  $p_H$  and to the dot 4 with the probability  $1 - p_H$ . The new probability of growth can be written as

$$\tilde{p}(i, k \rightarrow i', k') = \frac{p(i, k \rightarrow i', k')(1 - p_H(i, k, i', k')) + p(i, k \rightarrow i'', k'')p_H(i, k, i'', k'')}{\sum (p(i, k \rightarrow i', k')(1 - p_H(i, k, i', k')) + p(i, k \rightarrow i'', k'')p_H(i, k, i'', k''))} \quad (3)$$

where the point  $(i'', k'')$  is the neighboring point with respect to the point  $(i', k')$  in the clockwise direction with respect to the point  $(i, k)$  and the sum in the denominator refers to all possible growth processes. The probability 3 was used in computer simulations in place of the probability 2 with the same algorithm.

Let us turn to the probability  $p_H$  of deviation of the growth due to the magnetic field. In our model, during the process of growth, two constant forces act on charge carriers,  $F_L$  and  $F_H$  (Fig.1). When the resulting force is near the dot 3,  $p_H \rightarrow 1$ ; when the resulting force is near the dot 4,  $p_H \rightarrow 0$ . It is clear

that the probability  $p_H$  is proportional to  $\frac{F_L}{F_{loc}}$ . If  $F_{loc}(i, k, i', k') < F_C$  then the growth does not occur and  $p_H$  should be zero. If  $\frac{F_L}{F_{loc}} > 1$  then we let  $p_H = 1$ . The Lorentz force  $F_L$  is proportional to the velocity of charge carrier. In our model we can take into account only a local velocity, which arises in the first step of growth process due to the acceleration in the local field  $F_{loc}$ . So, the Lorentz force should be proportional to  $\sqrt{lF_{loc}(i, k, i', k')}$ .

Based on these considerations, we choose the probability of deviation of the growth due to the magnetic field,  $p_H$ , in the following form:

$$p_H(i, k, i', k') = \begin{cases} 0 & , \quad F_{loc}(i, k, i', k') < F_C \\ \frac{F_L(i, k, i', k')}{F_{loc}(i, k, i', k')} & , \quad \frac{F_L(i, k, i', k')}{F_{loc}(i, k, i', k')} \leq 1, \quad F_{loc}(i, k, i', k') \geq F_C \\ 1 & , \quad \frac{F_L(i, k, i', k')}{F_{loc}(i, k, i', k')} > 1, \quad F_{loc}(i, k, i', k') \geq F_C \end{cases} \quad (4)$$

where  $F_L(i, k, i', k') = \sqrt{lF_{loc}(i, k, i', k')}H$  and  $H$  is the value of the magnetic field.

In our computer simulations we consider a  $500 \times 500$  lattice. The solutions of the Laplace equation were obtained by the iteration method [11]. Before starting of each realization of growth we performed 20000 iterations and after each step of growth the number of iterations was 40. This procedure gives a good convergence. The number of particles in clusters was 9000.

The fractal dimension was calculated by the method described in [11]. For every realization we plotted the  $\log N(R)$  versus  $\log R$  where  $N(R)$  is the number of particles belonging to the structure and being within a circle of radius  $R$ . The fractal dimension is obtained by fitting a straight line to the data scaling region. For every set of the same parameters of the model ( $H, F_C, V_R, \eta$ ) we made about 100 realizations. Thus the statistical fluctuations were reduced.

Let us discuss the results of these calculations.

We start our simulations with the case of the zero magnetic field and the zero values of the parameters  $F_C$  and  $V_R$  in order to compare our results with the results of the different authors. In the Table.1 we present the dependence of the Hausdorff dimension  $D$  on the exponent parameter  $\eta$ .

$\eta$	$D$ (our results)	$D$ (according to [11])	$D$ (according to [13])
0.5	$1.89 \pm 0.02$	$1.89 \pm 0.01$	1.92
1	$1.73 \pm 0.02$	$1.75 \pm 0.02$	1.70
2	$1.6 \pm 0.03$	$\sim 1.6$	1.43

Table 1. Dependence of the Hausdorff dimension  $D$  on the exponent  $\eta$  used in the relation between probability and local field (Eq.2).

Our results are in a good agreement with the results, obtained earlier.

The example of the computer-generated discharge pattern (Lichtenberg figure [2]) corresponding to the following set of parameters:  $H = 0, \eta = 1, F_C = 0$  and  $V_R = 0$  is shown in the Fig.2. In the Fig.3 we show the computer-generated discharge pattern in the presence of the external magnetic field. The white lines in the figures 1 and 2 correspond to the leader channels. Unlike the pattern, presented in the Fig.2, the leader channels in the magnetic field are distorted and appear to have a circular shape outside the central electrode region. The direction of the bending corresponds to the movement of electrons in crossed electric and magnetic field. At the lower left corner of the Fig.3 we plotted the cyclotron orbit of an electron. One can note that the Larmor radius of an electron is about two orders of magnitude smaller than the radius of curvature of the leader channel.

The saturation of the fractal dimensionality, with growing magnetic fields, at the value of  $D = 1.67$ , which is one of the main results of this paper, is presented in Fig. 4. The plot starts from the value of  $D = 1.65$ , in the absence of magnetic field, which is smaller than the fractal dimensionality reported in [11].

Such difference results from the fact that in [11] the critical field value for the breakdown was not taken into account. We have improved their calculations by introducing the minimal value of the electric field for the breakdown between two successive points. In this case the breakdown pattern is more directionally selected, and a lower fractal dimensionality results. With the increase of the external magnetic field, the MFD growth and finally saturates at an universal for high magnetic fields values of  $1.67 \pm 0.01$ . The growth of MFD with the magnetic field could be expected, since the curved trajectories fill up the space more densely than the straight ones. The existence of an universal limit, however, is far from being obvious. Following the directed percolation models, one could think that the saturation of MFD will occur at  $D = 2$ . Fig 4. shows clearly that in this system the MFD saturates due to the physics of current carrying streamers.

An unexplained feature in the experiments of [3] is the 'spider-legs' form of the breakdown pattern in the absence of the external magnetic field. We outline here that the streamer currents are rather strong,  $10 \div 100$  A, and their influence on the streamer pattern can be very important. To describe this phenomenon we have taken into account the magnetic interaction between current carrying streamers, in the framework of the modified active walker model (MAWM) which is described in what follows. The results of our calculations, presented in Fig. 5, show that our model correctly describes the experimentally observed 'spider-legs' effect.

Active walker models have been used to describe different pattern formation problems [14, 15]. In these models the walkers movement is subject to the influences of the environment and vice versa. We describe the leader channel propagation in the magnetic field using the active walker model. The Lorentz force acting on the fast-moving electrons is particularly effective in the high-field regions in the leader tips, where the channel formation takes place.

Let us consider again a two-dimensional square lattice in which a central point represents one of the electrodes while the other electrode is modeled as a circle at large distance. The discharge starts at the central electrode, so initially several walkers are set in the vicinity of it. The walkers move in a potential which is the solution of the Laplace equation with the boundary conditions determined by electrodes and discharge structure. During a step of growth each walker moves. The solution of Laplace equation is found by iteration method [11] after each step of growth.

When a walker moves to a point, this point starts belonging to the breakdown structure. The potential of the central electrode is 0, the potential of a point in the structure is  $V_{i,k} = V_{l,m} + V_R * l$ , where  $V_{l,m}$  is the potential of a points from which growth go on,  $V_R$  is an internal field in the structure and  $l$  is 1 for bonds parallel to the grid and is  $\sqrt{2}$  for diagonal bonds. In the absence of the magnetic field, the probability of a walker step is a function of the local field  $F_{loc}$  [14]. The breakdown occurs only if the local field is greater then the critical field of growth  $F_C$ .

The magnetic field is taken into account by the following way. We add to the local field in the direction perpendicular to the previous move of the walker the Lorentz force which is proportional to the magnetic field. The magnetic field acting on the  $i$  walker is  $H_i = H_0 + H_I^i$ , where  $H_0$  is external magnetic field and  $H_I^i$  is the field created by currents of the breakdown structure. We calculate  $H_I^i$  by means of the Biot-Savart law:

$$H_I^i = \sum_{k \neq i, l} \frac{I}{r^3} d\vec{s}_{k,l} \times \vec{r} \quad (5)$$

where  $I$  is the current in a channel,  $d\vec{s}_{k,l}$  is  $l$  element of the  $k$  channel and  $\vec{r}$  is the vector pointed from the position of  $d\vec{s}_{k,l}$  to the position of the  $i$  walker.

The main results of this part of studies are presented in Fig. 5.

To summarize, we have generalized and modified the existing dielectric breakdown models to explain the experimental observations of the propagation of a streamer near an insulating surface under the influence of a transverse magnetic field. We have introduced the concept of the Magnetic Fractal Dimensionality (MFD) and have obtained its saturation with growing magnetic fields. The Universal Magnetic Fractal Dimensionality (UMFD) equals 1.670 which is superior to 1.65, the one in the absence of a magnetic field. Inclusion of the magnetic interaction between the current-carrying streamers results in the 'spider-legs' like streamer patterns at lower fields, which corresponds to the experimental observations.

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## Figure Captions

Fig.1.

Bonds connecting a lattice point with one of eight adjacent lattice points in a two-dimensional square lattice. The geometry of the experiment is represented by a central electrode at some point while the other electrode is a circle at large distance. The discharge starts at the central electrode and grows by one lattice bond per growth step.

Fig.2.

Computer-generated discharge pattern (Lichtenberg figure) corresponding to the following set of parameters:  $H = 0$ ,  $\eta = 1$ ,  $F_c = 0$  and  $V_R = 0$  as explained in the text. The white lines correspond to the leader channels.

Fig.2.

Computer-generated discharge pattern in the presence of the external magnetic field. The leader channels in the magnetic field are distorted and appear to have a circular shape outside the central electrode region, due to the action of the magnetic field.

Fig.4.

The saturation of the fractal dimensionality, with growing magnetic fields, at the value of  $D = 1.67$ .

Fig.5.

The 'spider-legs' form of the breakdown pattern following from the magnetic interaction between the streamer currents: a) current-current interactions are not taken into account,  $H = 0$ ; b) current-current interactions are taken into account,  $H = 0$ ; c)  $H \neq 0$ .



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